

The Postulate of Vacuum as the Medium of Matter Waves and the Extension of the Equivalence Principle

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The vacuum polarization described in quantum electrodynamics inspires the reconstruction of the relationship between a vacuum and a particle. It is reasonable to amalgamate vacuum and matter into one object: the metric space. Thus, the motion of matter can be described as the propagation of the state of space in a vacuum, the vacuum being the medium through which the matter wave propagates. The consequence of unifying a particle and vacuum with metric space is that all the properties of the particle should be described as the intrinsic properties of the space. By defining the affine curvature tensor symmetric part (electric field analogue) and an antisymmetric part (magnetic field analogue, zero divergence), the energy density is proposed to be proportional to the Kretschmann scalar, $u = c\hbar R^{ijkl} R_{ijkl}/2$, and the momentum similar to electromagnetic momentum density $\epsilon_0 E \times B$; a metric with a curvature proportional to m/r^2 is thus obtained. The equivalence of the torsion of the affine connection space with the angular momentum of matter is then discussed. The quantized spin angular momentum eigenstate in quantum mechanics is related to a connected torsional manifold. In the case of $\hbar/2$ spin, it corresponds to a Möbius circle. It is remarkable that the two classified elementary particles, bosons and fermions, correspond to the two types of topological manifolds, orientable and non-orientable manifolds.

The reinterpreted concept of matter, in the form of curved space, gives rise to the idea of absolute space. The time dilation observed in the Global Positioning System, as predicted by special relativity and depending on the velocity in relation to the "universal frame," provides evidence of absolute space.

Introduction

"The question is, of course, is it going to be possible to amalgamate everything, and merely discover that this world represents different aspects of one thing?" After Feynman described the amalgamation process that had happened in physics, he described a simple and beautiful picture of the world that might be [1].

Today, matter is considered to be composed of elementary particles, which are classified into quarks, leptons, and mesons. These particles are described by a field theory known as the $U(1) \times SU(2) \times SU(3)$ Standard Model. The critical aspect is that these fundamental particles can transform into one another under specific conditions. It is natural to consider that there are more fundamental objects, such as superstrings. However, it is difficult to determine how to validate these theoretical concepts.

Currently, a dichotomy exists in theoretical physics. Strong, electromagnetic, and weak interactions are described by quantum field theory. These fields reside in flat and rigid space-time, whereas the gravitational field is defined as deformed space-time according to Einstein's theory. Although a quantum field appears separate from space, certain basic phenomena, such as vacuum polarization, reveal the inner relationship between particles and vacuum. Amalgamating particles and a vacuum into one concept, metric space, that is, stating that a particle is also equivalent to deformed space, would be a reasonable idea. Thus, we can obtain a picture that Feynman anticipated.

The Reinterpreted Concepts of Space, Time, and Matter

Space, time, and matter are foundational concepts in physics. Intuitively, matter occupies space, which means that matter has the same basic property, length, as space. The motion of matter, measured by the change in the distance between two matter objects, is a basic phenomenon that connects these three concepts in classical mechanics. In contrast, the opposite idea of motion is that of wave propagation. Maxwell believed in the existence of a medium for electromagnetic waves, similar to other waves, known as the luminiferous ether. However, the Michelson-Morley experiments that attempted to determine the absolute velocity of the Earth through the hypothetical ether found no expected result. Lorentz explained the negative result with the assumption that all bodies shortened their dimension in the direction of motion by a factor of $\sqrt{1 - u^2/c^2}$ [2,3]. However, Einstein provided a different explanation by proposing the special theory of relativity based on the principle of relativity and the constancy of the velocity of light [4]. According to the special theory of relativity, space, time, and mass all change along with the velocity of matter:

$$l = l_0 \sqrt{1 - u^2/c^2} \qquad t = t_0 / \sqrt{1 - u^2/c^2} \qquad (2.1)$$

$$m = m_0 / \sqrt{1 - u^2/c^2} \quad (2.2)$$

This equation demonstrates that the properties of space, time, and matter are not independent; rather, they are interrelated through the common parameter of velocity. However, the concept of time is commonly considered with space to be an entity of four-dimensional space-time because of the Lorentz transformation, as follows:

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}, \quad y' = y, \quad z' = z \quad (2.3)$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \quad (2.4)$$

However, it should instead be considered an observer effect. No actual physical effect verifies the fusion of space and time, whereas Eq. (2.1) represents changes in the intrinsic properties of space and time.

Selleri proposed an alternative space-time transformation based on noninvariant one-way speed of light considerations in a series of papers [5-7]; the time transformation is different from Eq. (2.3) as follows:

$$t' = t \sqrt{1 - u^2/c^2} \quad (2.5)$$

This equation implies that the time dilation effect is independent of spatial position, thus supporting absolute clock synchronization and suggesting the existence of a privileged frame. Furthermore, following this theory, Buonaura developed the inertial transformation of the electric and magnetic fields [8]. However, the theory does not explicitly indicate a privileged frame of reference.

Wave propagation is a phenomenon that seems to be mutually exclusive from matter motion. A particle is indivisible, whereas a wave is divisible. The debate over whether light behaves as a particle or a wave began with Newton and Huygens. After Maxwell integrated the optics into electromagnetic wave theory, there is no further argument. However, the study of blackbody radiation, which classical electromagnetic theory could not explain, led Planck to suggest a hypothesis regarding the quantization of energy. Building on this hypothesis, Einstein proposed a return-to-particle theory to explain the photoelectric effect. Physicist de Broglie went a step further with the hypothesis, suggesting that material particles, just like photons, can have a wave-like aspect [9,10]. This generalization led to the development of wave mechanics, which describes and explains natural phenomena. However, the concept of a medium for matter waves is no longer accepted.

Quantum mechanics offers a description that differs from classical mechanics. It reveals the intrinsic property and inner relationship of space, time, and matter. Classical mechanics abstracts a material particle as a point with a position and velocity at a time instant. In contrast, quantum mechanics describes the particle's state as a space distribution with a

complex value at various points and the wave function $\psi(r, t)$, which contains all the information about the particle. This complex value can also be considered a two-dimensional vector of the subspace of the three-dimensional tangent space of the physical space. This perspective is valuable for understanding the relationship between quantum theory and general relativity. The characteristic value of matter is represented by the operation on the quantum state, especially for energy, momentum, and angular momentum operations:

$$E = i\hbar \frac{\partial}{\partial t} \quad (2.6)$$

$$P_x = \frac{\hbar}{i} \frac{\partial}{\partial x}, \quad P_y = \frac{\hbar}{i} \frac{\partial}{\partial y}, \quad P_z = \frac{\hbar}{i} \frac{\partial}{\partial z} \quad (2.7)$$

$$L_z = \frac{\hbar}{i} \frac{\partial}{\partial \varphi} \quad (2.8)$$

The operations above are in the form of mathematical derivatives. They represent the characteristic asymmetry of space and time in a matter system. The angular momentum L_z is related to the asymmetry of the space rotation along the z-axis. The momentum P_x is associated with the asymmetry of the spatial transport in the x-direction, and the energy of matter is related to the asymmetry of the time of the system. The operators of the energy and momentum have dimensions of $\hbar T^{-1}$ and $\hbar L^{-1}$, respectively, while the dimension of the angular momentum is $\hbar L^0$. The dimension of momentum density is $\hbar L^{-4}$. With the conversion of time and space using the velocity of light c , the energy can also be expressed as $c\hbar L^{-1}$. Thus, the energy density is $c\hbar L^{-4}$. These dimensional expressions led to several conclusions, as described later in this document.

Considering Schrödinger's wave equation

$$i\hbar \frac{\partial}{\partial t} \psi(r, t) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(r, t) \quad (2.9)$$

This equation demonstrates that the asymmetry of space determines the asymmetry of time, indicating that the property of time is not independent of space. In contrast, the conservation of energy, momentum, and angular momentum comes from the symmetry of the system under translations in time, space, and spatial rotations. Therefore, energy, momentum, and angular momentum are different manifestations of the intrinsic property of space-time.

Another fundamental concept in quantum mechanics is the concept of probability. The wave function $\psi(r, t)$ characterizing the quantum state is interpreted as a probability amplitude of the particle's presence. In classical mechanics, the state of the system is known at all times if its initial state is known. Knowing the state of the system, one can predict with certainty the result of any measurement performed at time t . Quantum mechanics offers a radically different perspective on the world. The concept of a stochastic event is more fundamental than the concept of time, which measures changes in nature. It is expected that time will ultimately be expressed as a function of the number of stochastic events.

Statistical mechanics provides a unique perspective on space, time, and matter. In a multi-particle system, the number of ways in which particles are arranged in space represents the disorder of the system. The logarithm of this number is the entropy. According to the law of thermodynamics, the total entropy of the isolated system always increases over time. Therefore, an increase in entropy characterizes the direction of time. Additionally, the corrected calculation of the number of ways particles are arranged in space relies on two key assumptions.

1. The exchange of two of the same type of particles gives the same state of the system.
2. The exchange of two volume elements of a system's divided space gives a different state of the system.

These two assumptions are contrary to our intuitions. Intuitively, there is no characteristic for distinguishing different volume elements, whereas matter objects, as substances in the world, should be distinguishable from each other. A more accurate explanation for these two assumptions is that space is the substance of a world that possesses distinct characteristics. In contrast, matter is a state of space, also called information, and two identical states are indistinguishable.

The most profound relationship between space and matter is revealed by vacuum fluctuations in quantum field theories, which create pairs of particle-antiparticles with opposite energies from the vacuum. These pairs would then annihilate into the vacuum after a moment. Even in classical physics, the special relativity energy-momentum equation has two solutions:

$$E^2 = P^2 c^2 + m_0^2 c^4 \quad (2.10)$$

$$E = \pm \sqrt{p^2 c^2 + m_0^2 c^4} \quad (2.11)$$

This equation demonstrates that energy can possess both positive and negative values, and that there are situations in which the positive and negative energies cancel each other out when combined. The corresponding relativistic wave function of a free particle, the Dirac Equation, is

$$i\hbar \frac{\partial}{\partial t} \psi(r, t) = (-i\hbar c \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \alpha_4 \cdot m_0 c^2) \psi(r, t) \quad (2.12)$$

The solutions of this equation automatically contain all the possibilities of virtual (and real) pair formation and annihilation, along with ordinary scattering processes [11]. The spontaneous mutual transformation of virtual pairs and a vacuum has a real effect known as the vacuum polarization effect. A background electromagnetic field polarizes the disordered virtual electron-positron pairs, which in turn modifies the electrical potential. In atomic hydrogen splitting between $2S_{1/2}$ and $P_{1/2}$, the vacuum polarization effect response

is a correction of 27 MHz in the Lamb shift [12]. The idea of the annihilation and creation of matter imposes the reinterpretation of the concept of matter.

The common opinion is that a substance cannot be created and destroyed; if matter is not persistent, what is persistent? A reasonable idea is to amalgamate matter and vacuum into one thing, the metric space, because matter and vacuum are different states of physical space that should be considered the only substance of the world. A principle for the substance and the state is that the substance is distinguishable, whereas the state is indistinguishable. Matter represents an asymmetrical state of space, whereas a vacuum is the state of space with the most symmetry. Under this assumption, matter motion is the propagation of the asymmetrical state of space. The three-dimensional physical space is the only substance in our world.

In mathematics, the deformation of space is expressed through the concept of a manifold. Two differential forms on the manifold encode information about how the space deforms. One is the metric form, which describes the distance between two infinitesimal neighboring points and is expressed as

$$G \equiv ds^2 = g_{ij}dx^i \otimes dx^j \quad (2.14)$$

Another differential form is the connection on a vector bundle that describes the relationship between the tangent spaces of two infinitesimal neighboring points and a matrix in $GL(3)$, which is expressed as

$$\nabla \mathbf{e}_i = \Gamma_{ij}^k \mathbf{e}_k dx^j \quad (2.15)$$

The two differential forms satisfy the compatible condition:

$$\nabla G = 0 \quad (2.16)$$

Thus, the lengths of the vector are preserved, and the structure group is $O(3)$. In differential geometry, tensors are used to describe the intrinsic properties of the manifold. Two important tensors are constructed from differential forms, namely, the curvature tensor R and the torsion tensor T :

$$R = R_{ijkl} du^i \otimes du^j \otimes du^k \otimes du^l \quad (2.17)$$

$$T = T_{ik}^j \frac{\partial}{\partial u^j} \otimes du^i \otimes du^k \quad (2.18)$$

The curvature tensor defined from the affine connection is called the affine curvature tensor R_{ijkl}^∇ , whereas one determined from the metric is called the Riemannian curvature tensor R_{ijkl} .

Despite these local properties that describe points of an infinitesimal neighborhood, various global properties of manifold objects have been revealed in terms of topological structures, which would correspond to the property of discreteness of the particles. It is expected that the topological three-dimensional manifold of physical space will explain more relationships between particles.

It is essential to distinguish between physical space and mathematical space. Differential geometry describes the static deformation of space. Matter propagation in nature requires that the state of space in one position can influence the state of space in another position, indicating that physical space is dynamic. Quantum mechanics reveals that the dynamic process of the universe is stochastic. It is anticipated that a dynamic manifold theory based on stochastic events will be constructed. Additionally, physical space possesses specific constant parameters that characterize the universe, such as the velocity of light (c), Planck's constant (h), the gravitational constant (G), and the elementary charge (e). The definition of distance between two positions in mathematics provides a measurable value in physics, which serves as the foundation for a physical space that a mathematical space can represent. In any case, differential geometry provides various models for describing the deformation of physical space.

Extension of Equivalence of Curvature Space to Its Source

Based on the equality of gravitational mass and inertial mass, Einstein established an equation that described how matter causes space-time to curve [13]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (3.1)$$

Where both the Ricci tensor $R_{\mu\nu}$ and curvature scalar R were constructed from the Riemannian

curvature tensor R_{ijkl} . This equation describes how the space-time metric deforms when matter is present. In other words, the gravitational field is equivalent to the curved space-time. Eq. (3.1) is the extension of Newton's gravitational law:

$$f = \frac{GmM}{r^2} \quad (3.2)$$

Newton's gravitational theory, although it lacks the term of moving matter (v/c) as one of the gravitational sources, can be compared with electromagnetic interaction. In electromagnetism, a moving charge produces a magnetic field with $\nabla \times cB = (v/c) \rho/\epsilon_0$, in addition to making an electric field with $\nabla \cdot E = \rho/\epsilon_0$.

Besides the curvature tensor in Einstein's theory, which replaces the electromagnetic field in electrodynamics, there are two critical differences between gravitational and electromagnetic interactions. Weinberg pointed out that "Maxwell's equations are linear

because the electromagnetic field does not carry charge, whereas gravitational fields carry energy and momentum and must therefore contribute to their own source" [14]. The other difference is that the gravitational interaction between two positive masses is attractive, enhancing both the kinetic energy and field energy. Hence, the energy of the gravitational field is contrary to its source energy.

The electrodynamic equation provides the formula for the energy density and momentum density of the electromagnetic field as follows:

$$u = \frac{\epsilon_0}{2} E \cdot E + \frac{\epsilon_0 c^2}{2} B \cdot B \quad (3.3)$$

$$g = \epsilon_0 E \times B \quad (3.4)$$

Thus, the total energy and momentum of the electromagnetic field, along with the charged matter, are conserved. However, several formulas for the energy and momentum of the gravitational field based on Einstein's gravitational theory are not tensors; hence, they are not satisfied [15,16].

Herein, this is considered differently. There are two requirements for the form of the energy density of gravitation:

1. The density should be a function of the curvature tensor of the three-dimensional space; the curvature tensor is the expression of the intrinsic property of curvature space in mathematics.
2. The magnitude of energy density of the gravitational field should be proportional to $1/r^4$. It is helpful to investigate the characteristics of the curvature tensor. Considering the curvature tensor R_{ijkl} as the second derivative, the dimension of the invariant curvature components is L^{-2} .

Additionally, the curvature tensor R_{ijkl} of the Riemannian manifold satisfies the following properties:

1. $R_{ijkl} = -R_{jikl} = -R_{ijlk}$,
2. $R_{ijkl} = R_{klij}$.
3. $R_{ijkl} + R_{iklj} + R_{iljk} = 0$. (For a 3-dimensional manifold, this equation is redundant)

An important theorem noted that "the curvature tensor of a Riemannian manifold M at a point p is uniquely determined by the sectional curvatures of all the 2-dimensional tangent subspaces at p " [17], meaning that the three components R_{1212} , R_{2323} , and R_{3131} determine the Riemannian curvature tensor. The R_{3112} , R_{1223} , R_{1223} are projection of R_{1212} , R_{2323} , and R_{3131} . For example, for a Riemannian curvature tensor with no zero component $R_{1212} = 1$,

under the rotation around the axis- u^1 with angle θ , no zero component with:

$$R'_{3112} = \cos \theta \sin \theta, \quad R'_{1212} = \cos^2 \theta, \quad R'_{1313} = \sin^2 \theta.$$

By selecting proper coordinates, R_{3112} , R_{1223} , and R_{2331} can all be made to vanish.

Now, considering the affine curvature tensor R^∇_{ijkl} of a manifold with torsion, and metric

compatible with the affine connection, the equivalent equation $R^\nabla_{ijkl} = R^\nabla_{klij}$ does not hold.

However, we can define the symmetric part of the affine curvature tensor (metric compatible)

$$R_{ijkl} \equiv \frac{1}{2}(R^\nabla_{ijkl} + R^\nabla_{klij}) \quad (3.5)$$

and the antisymmetric part as

$$C_{ijkl} \equiv \frac{1}{2}(R^\nabla_{ijkl} - R^\nabla_{klij}) \quad (3.6)$$

(where "C" stands for Cartan). The affine curvature tensor R^∇_{ijkl} is the sum of two parts:

$$R^\nabla_{ijkl} = R_{ijkl} + C_{ijkl} \quad (3.7)$$

There are three essential properties for two parts. The first is that under a rotation of coordinates of the tangent space, they are closed.

$$R'^\nabla_{mnop} = O_m^i O_n^j O_o^k O_p^l R^\nabla_{ijkl},$$

$$R'_{mnop} = O_m^i O_n^j O_o^k O_p^l R_{ijkl}, \quad (3.8)$$

$$C'_{mnop} = O_m^i O_n^j O_o^k O_p^l C_{ijkl}.$$

The two parts do not blend under the rotation of the coordinate. The other one is that the inner product of the affine curvature is equivalent to the sum of the inner products of each one.

$$|R^\nabla_{ijkl}|^2 = |R_{ijkl}|^2 + |C_{ijkl}|^2 \quad (3.9)$$

Rotating the coordinate around the axis u^3 with angle θ , for C_{ijkl} , there are

$$C'_{2331} = C_{2331},$$

$$C'_{3112} = C_{3112} \cos \theta + C_{1223} \sin \theta,$$

$$C'_{1223} = -C_{3112} \sin \theta + C_{1223} \cos \theta.$$

The three independent components act as a vector under coordinate rotation. Thus, we can define the divergence of C_{ijkl} . By threefold Hodge duality, this type-(0,4) tensor correspond to a vector,

$$B_{mn} = \varepsilon_{ijm} \varepsilon_{kln} C_{ijkl},$$

$$H_p = \varepsilon_{mnp} B_{mn}.$$

The vector H , being dual to an antisymmetric tensor, the same as the magnetic field, can also represent a curl form:

$$H = \nabla \times \omega. \quad (3.10)$$

Where ω is a torsion-related 1-form. Being a direct result of the closedness of exterior differentiation (Poincaré's lemma), we have:

$$\nabla \cdot H = 0. \quad (3.11)$$

Actually, these two parts have different geometrical meanings: the R_{1212} , R_{1313} , and R_{2323} indicate that the vector rotation plane is aligned with the displacement plane, whereas C_{3112} , C_{1223} , and C_{2331} indicate that the vector rotation plane differs from the path plane.

The curvature tensor R_{ijkl} , as the second derivative, has curvature components with dimensions that depend on the coordinate selection; however, the Gauss curvature is invariant as L^{-2} , making it convenient to define the Gauss form of curvature components.

$$K_{ijij}(E) = \frac{R_{ijij}}{G_{ij}} \quad (3.5)$$

$$K_{ijjk}(B) = \frac{C_{ijjl}}{\sqrt{G_{ij}}\sqrt{G_{jk}}} \quad (3.6)$$

Where $G_{ij} = g_{ii}g_{jj} - (g_{ij})^2$ is the determinant of the metric of the subspace. The dimensions of the inner product $R^{ijkl} R_{ijkl}$ are L^{-4} ;

Considering the formulas for the energy and momentum density of an electromagnetic field, it is valuable to define E' and B' based on E and B to obtain the symmetry formula for the energy and the momentum density as follows:

$$E' = \sqrt{\frac{\epsilon_0}{c}} E \quad B' = \sqrt{c\epsilon_0} B \quad (3.6)$$

Thus, Eq. (3.3) and Eq. (3.4) become:

$$u = \frac{c}{2} (E' \cdot E' + B' \cdot B') \quad (3.7)$$

$$g = E' \times B' \quad (3.8)$$

All the dimensions of E' and B' are $\hbar^{-1/2} L^{-2}$. Hence, neglecting the constant factor, it is possible to establish the corresponding relationship between the components of the curvature tensor and the electromagnetic field as follows:

$$\begin{aligned} K_{1212} &\rightarrow E'_z & K_{3131} &\rightarrow E'_y & K_{2323} &\rightarrow E'_x \\ K_{3112} &\rightarrow B'_x & K_{1223} &\rightarrow B'_y & K_{2331} &\rightarrow B'_z \end{aligned} \quad (3.9)$$

The Schwarzschild Solution for a static gravitational field indicates that K_{1213} , K_{2123} , and K_{3132} vanish, while K_{1212} , K_{1313} , and K_{2323} do not. We denote similar vectors K_E and K_B as two parts of the curvature tensor.

Analogous to the dimensionless fine-structure constant $\alpha = e^2/\hbar c$, which measures the strength of the electromagnetic force, with the use of the notation Planck mass

$$M_{Pl} = \left(\frac{\hbar c}{G}\right)^{1/2} \cong 1 \times 10^{-8} kg \quad (3.10)$$

And the dimensionless value

$$\beta = \frac{m}{M_{Pl}} \quad (3.11)$$

measuring the strength of the gravitational force. For the known mass of particles, β is significantly less than 1. Thus, the value

$$\gamma = \frac{m/M_{Pl}}{r^2} \quad (3.12)$$

is proportional to the strength of the static gravitational field of mass m and has dimensions of L^{-2} . The integral in volume of the square of γ gives the energy

$$E = \hbar c \int \gamma^2 dVol \quad (3.13)$$

The factor \hbar and c canceled out in this equation. Thus, analogous to the electromagnetic energy density, we propose the energy density of the curvature space in terms of the curvature tensor component:

$$u = \frac{c\hbar}{2} (K_E \cdot K_E + K_B \cdot K_B) \quad (3.14)$$

The formula of momentum density is analogous to that of electromagnetism.

$$g = \hbar (K_E \times K_B) \quad (3.17)$$

The electromagnetic field is equivalent to a form of curvature tensor with a physical foundation. The electromagnetic field can be considered as an average effect of photons. Considering a photon moving in a gravity field from the position of the energy state E_1 to E_2 (even for the case of the gravitational interaction between two photons), the frequency of this photon would change from ω_1 to ω_2 , corresponding to the energies $\hbar\omega_1$ and $\hbar\omega_2$. This phenomenon can be explained by the relativity principle or through the conservation of energy, in that energy is transferred between a gravitational field and an electromagnetic field. It can also be understood as a redistribution of energy in space. Therefore, the energy itself is an intrinsic property of curved space, and a photon is essentially a manifestation of curved space that has specific topological properties. By obtaining the expressions for the energy and momentum density of a gravitational field, the energy and momentum density of matter can be generalized, and the source of gravity itself can also be expressed as the curvature tensor.

The Schwarzschild Solution's metric for a static and isotropic gravitational field in Einstein's equation is:

$$d\tau^2 = [1 - \rho/r]dt^2 - [1 - \rho/r]^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 \quad (3.19)$$

Where $\rho = 2GM/c^2$, and the spatial metric is

$$ds^2 = [1 - \rho/r]^{-1}dr^2 - r^2d\theta^2 + r^2\sin^2\theta d\varphi^2 \quad (3.20)$$

The curvature tensor components for this metric are proportional to $1/r^3$ but not $1/r^2$. However, considering that the gravitational field corresponds to the curvature component

for a static and spherically symmetric gravitational field, the $K_{\theta\varphi\theta\varphi}$ should be proportional to $1/r^2$, with $K_{r\varphi r\varphi}$ and $K_{r\theta r\theta}$ being zero. Calculations show that the metric:

$$ds^2 = dr^2 + (1 + m/M_{Pl})r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (3.21)$$

satisfies these requirements, and:

$$K_{\theta\varphi\theta\varphi} = \frac{m/M_{Pl}}{r^2} \quad (3.22)$$

As $r \rightarrow \infty$, $K_{\theta\varphi\theta\varphi} \rightarrow 0$, indicating flat space.

Substituting mass m with $-m$ in Eq. (3.22), we obtain:

$$ds^2 = dr^2 + (1 - m/M_{Pl})r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (3.23)$$

Here, the metric space is assumed to be unchanged in the r -direction and to contract in θ and φ -direction for sources of positive energy and expand for sources of negative energy. Owing to the contradiction between the gravitational field energy and its source energy, positive and negative energy can be expressed as compacting and expanding properties of the metric space, respectively. Thus, vacuum polarization can be described as the symmetry breaking of a flat space into a compact and an expanded metric space.

Curvature Superposition

Superposition is a fundamental concept in physics and mathematics, particularly in the fields of wave theory and quantum mechanics. It applies notably to electric and magnetic fields. While electric potential conveniently describes the interaction of two static electric charges, the more fundamental concept is the superposition of electric fields. The change in the spatial integral of electric field energy when their distance changes is:

$$\Delta U = \int \frac{1}{2} (\mathbf{E}_1 + \mathbf{E}_2)^2 dV - \int \frac{1}{2} (\mathbf{E}'_1 + \mathbf{E}'_2)^2 dV = \int \mathbf{E}_1 \cdot \mathbf{E}_2 dV - \int \mathbf{E}'_1 \cdot \mathbf{E}'_2 dV \quad (4.1)$$

The volume integral yields the familiar potential energy formula:

$$\int \mathbf{E}_1 \cdot \mathbf{E}_2 dV = q^1 q^2 \int_a^\infty \frac{dr}{r^2} = \frac{q^1 q^2}{a} \quad (4.2)$$

The curvature tensor, as a multilinear function on the tangent space of a manifold, applies linear combinations. Eq. (3.23) shows that the curvature component is a linear function of mass m ; thus, multiplying mass m will multiply the curvature

$$K_{ijij}(m_1 + m_2, x) = K_{ijij}(m_1, x) + K_{ijij}(m_2, x) \quad (4.3)$$

Another condition is that masses produce the combination of curvature components at different points. We notice that the two parts of curvature act like vectors under coordinate rotation, and the inner product of the two parts of curvature tensor

$$\langle K_E, K_E \rangle = K^{1212} K_{1212} + K^{1313} K_{1313} + K^{2323} K_{2323} \quad (4.4)$$

$$\langle K_B, K_B \rangle = K^{3112} K_{3112} + K^{1223} K_{1223} + K^{2331} K_{2331} \quad (4.5)$$

are invariant under coordinate transformations. We also notice that the geometric interpretation of curvature component is an oriented rotating angle during parallel displacement along an oriented closed curve; this is canceled out when two contrary oriented angles are considered. Thus, the curvature components of the two parts combine to act as vectors for different mass sources.

$$K_{ijkl}(\mathbf{r}) = K_{ijkl}(m_1, \mathbf{r}_1 - \mathbf{r}) + K_{ijkl}(m_2, \mathbf{r}_2 - \mathbf{r}) \quad (4.6)$$

We assume that the inner product of curvature, multiplied by the physical constant $\hbar c$, is equivalent to the energy density; thus, the total energy of the gravitational field can be expressed as the total spatial integral of the inner product K_{ijkl} . For a static and spherically

symmetric gravitational field, K_E takes a form similar to the static electric field, following the same superposition principle. The gravitational interaction energy of two static masses is

$$U = \hbar c \int K_E(x) \cdot K_E(x) dV \quad (4.7)$$

The primary difference from a static electric field is that the volume depends on the curvature. For a weak static gravitational field in normal coordinates, the metric determinant is

$$\sqrt{g} \approx 1 \quad (4.8)$$

Thus, we have a similar result as Eq. (4.2)

$$\hbar c \int K_E(x) \cdot K_E(x) dV \approx \frac{GmM}{d} \quad (4.9)$$

This equation demonstrates the superposition of the K_E of static gravitational fields. In general cases, the superposition principle should also apply to the K_E and K_B .

The Torsion Tensor of Manifolds Corresponding to The Spin Angular Momentum of Particles

The extensional equivalence principle posits that all properties of a particle must be represented as properties of the deformational space itself. A more generalized manifold, compared to the Riemannian manifold used by Einstein for a gravitational field, an affine connection manifold provides space with greater variability. Torsion, the antisymmetric part of a general affine connection, was introduced by Élie Cartan [18-20] and is defined as

$$T_{ij}^k = \Gamma_{ij}^k - \Gamma_{ji}^k \quad (5.1)$$

He recognized the tensor character of torsion and developed a differential geometric formalism for it.

He was also the first person to connect the torsion of space-time with the intrinsic angular momentum of matter. In fact, he hypothesized that the spin angular momentum was the source of the gravitational field. This modified gravitational theory is also called the Einstein-

Cartan Theory. However, the generalized equivalence principle provides a different perspective. The hypothesis that matter is equivalent to deformational space means that the spin angular momentum of a particle is comparable to the torsional property of space.

Considering the wave function $\psi(r, t)$ of quantum mechanics as a two-dimensional vector field of the tangent space would lead to a direct relationship with manifold theory. The L_z angular momentum formula of quantum mechanics in spherical (or polar) coordinates is expressed in Eq. (2.6), and its eigenfunction is

$$\phi(\varphi) = e^{im\varphi} \quad (5.2)$$

which can be expressed in the form of the sum of the real number part and imaginary number part as a two-dimensional vector:

$$\phi(\varphi) = \cos m\varphi + i \sin m\varphi \quad (5.3)$$

From a quantum mechanics point of view, because a wave function must be continuous at all points in space, it requires

$$\phi(\varphi + 2\pi) = \phi(\varphi) \quad (5.4)$$

Therefore, m is an integer. It is commonly considered that m is only an orbital angular momentum eigenvalue, whereas the eigenvalue of the spin angular momentum can be an integral or a half-integral. However, the electron's orbital angular momentum being an integral may be a consequence of the spin of a photon. Considering the complex value of the wave function as a vector of the tangent space of the physical three-dimensional space and $\varphi \rightarrow \varphi + 2\pi$ as the vector's parallel transport, the manifold remains continuous when $\theta(\varphi + 2\pi) = -\theta(\varphi)$. This case applies to non-orientable manifolds. A Möbius circle occurs when $m = 1/2$ (Fig. 1).

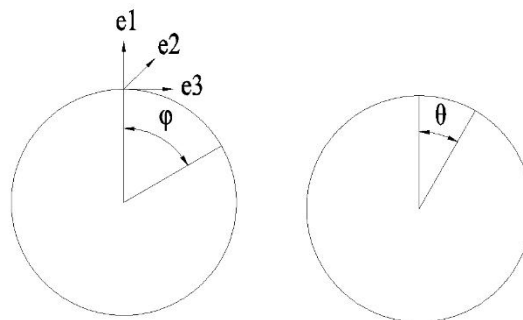
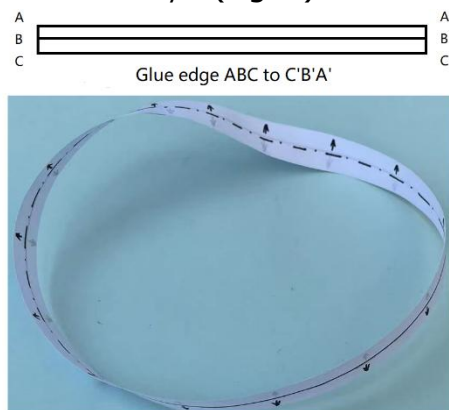


Fig.1 The Möbius circle, a non-orientable manifold, model for $1/2$ spin. Fig.2 Torsion, characterizing a twist of a moving frame around a circle.

Of course, models of other types of angular momentum can be constructed by gluing edge ABC to A'B'C' or ABC to C'B'A' after twisting edge A'B'C' by an integral or half-integral multiple of 2π .

In differential geometry, the twist of the manifold responds to the torsion tensor. A connection can be decomposed into a symmetric (torsion-free) part and an antisymmetric

(torsion) part:

$$\Gamma_{ij}^k = -\frac{1}{2}T_{ij}^k + \tilde{\Gamma}_{ij}^k \quad (5.5)$$

Where

$$\tilde{\Gamma}_{ij}^k = \frac{1}{2}(\Gamma_{ij}^k + \Gamma_{ji}^k) \quad (5.6)$$

When selecting the geodesic normal coordinate, the symmetric part $\tilde{\Gamma}_{ij}^k$ vanishes; thus, only the torsion tensor of the connection remains. Considering the three orthogonal unit vectors \mathbf{e}_1 , \mathbf{e}_2 , and \mathbf{e}_3 , parallel transport is performed along the geodesic. Let \mathbf{e}_3 be the tangent vector of a curve. Because the connection is compatible with the metric space, the angle between any two of the three vectors is preserved. As a result of the geodesic equation, the antisymmetric part of the connection does not affect the parallel displacement of \mathbf{e}_3 , whereas the other two vectors, \mathbf{e}_1 and \mathbf{e}_2 , rotate around the axis of \mathbf{e}_3 with the same angle by torsion. With the Möbius circle being geodesic, the two vectors \mathbf{e}_1 and \mathbf{e}_2 twist the angle θ of π when \mathbf{e}_3 rotates along the circle with an angle φ of 2π (Fig.2). The rates of the two angles correspond to the physical angular momentum.

$$m = \frac{d\theta}{d\varphi} \quad (5.7)$$

Thus, in quantum mechanics, the eigenvalue of L_z is the torsion of the manifold, and the close property of the manifold represents the global property of a physical system.

It is significant that the closed strip B-B' in Fig. 1 with a different twist has two different topological features of the manifold. One is orientable when the twist value m is equal to an integer, and the other is non-orientable when this value is equal to a half-integer. All elementary particles are divided into two types based on spin: bosons with integer spin and fermions with half-integer spin. It is natural to consider two types of particles corresponding to two types of manifolds. Bosons and fermions follow different statistical laws. It is expected that the topological feature of space will result in a different method for superposition, leading to distinct statistical methods.

In quantum mechanics, angular momentum is classified into two types: orbital angular momentum and spin angular momentum. The angular momentum operators are expressed as spatial operators and matrix operators. By interpreting the complex value of the wave function as a two-dimensional vector of the tangent space, spin 1/2 has a Möbius circle topology structure. The Spin angular momentum can also be expressed as a spatial operator. Certainly, spherical harmonics $Y_l^m(\theta, \varphi)$ for half-odd-integer values of l and m exist [21]. In the Dirac Equation Eq. (2.10), m_0 is a number, but not an operator. If m_0 is expressed as an operator, denoted by $\mathbf{M}(\mathbf{r})$, for a rest massive particle, the wave equation can be formulated as follows:

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \mathbf{M}(\mathbf{r}) \psi(\mathbf{r}, t) \quad (5.8)$$

A rest particle has spherical symmetry, and the spherical polar coordinate $\mathbf{M}(\mathbf{r})$ can be expressed as the product of a function of r and the square of the angular momentum:

$$\mathbf{M}(\mathbf{r}) = R(r) \mathbf{L}^2(\theta, \varphi) \quad (5.9)$$

This angular momentum is responsible for the spin angular momentum and has a $1/2$ eigenvalue of l and m with the eigenfunction:

$$\phi(\theta, \varphi) = \frac{1}{\pi} \sqrt{\sin \theta} e^{i\varphi/2} \quad (5.10)$$

Thus, the angular momentum in quantum mechanics is related to the geometric torsion structure of space.

Time Dilation in Global Positioning System Supports the Concept of Absolute Space

The postulate that vacuum and matter should be unified into one concept of metric space has the consequence that space is the substance of the world. The motion of matter is simply the propagation of a state of space, which is an internal relevant frame. The change in the velocity of matter is the result of a change in the curvature of matter. Thus, experimentally verified relativity effects, such as time dilation and mass increase, should not be considered as observed effects of different reference frames [22,23]. The above internal relevant frame applies to Newton's concept of absolute space. In his words, "Absolute space, in its own nature and with regard to anything external, always remains similar and unmovable" [24]. A possible rest reference frame is the "mean rest frame of the universe" in which the cosmic microwave background is isotropic. The velocity of the Sun in relation to this "universal frame" measured from the dipole cosmic microwave background is approximately 370 km/s, with the direction of the galactic coordinate system at $l \approx 264^\circ$, $b \approx 48^\circ$. However, special relativity states that all inertial reference frames are equivalent. The difference between the results for these two ideas can be examined experimentally. The motional time dilation effect of clocks in the Global Positioning System (GPS) provides a valuable means of experimentally verifying these two principles. For the concept of absolute space, there is only one reference frame in which $\Delta t = \Delta t_0 / \sqrt{1 - u^2/c^2}$ is valid.

GPS utilizes accurate, stable atomic clocks in its satellites, and the 24 satellites are distributed across six orbital planes at an inclination of 55° in near-circular orbits with a period corresponding to 0.5 sidereal days. The principle of position determination and time transfer in the GPS can be stated as follows: four signals from position \mathbf{r}_j at time t_j ($j = 1, 2, 3, 4$) are received at position \mathbf{r} at the same instant t , according to the principle of the constancy of the speed of light.

$$c^2(t - t_j)^2 = |\mathbf{r} - \mathbf{r}_j|^2, \quad j = 1, 2, 3, 4 \quad (6.1)$$

where $c = 299792458$ m/s. By solving these four equations for space-time coordinates, the $\{\mathbf{r}, t\}$ of the reception event can be confirmed. Because of motional and gravitational frequency shifts in clocks, the local time of four distant clocks should be adjusted to synchronize at the specified reference frame so that Eq. (6.1) is valid. Timing errors of 1 ns result in positioning errors of approximately 30 cm. However, this leads to a serious error when asserting Eq. (6.1) in the Earth-centered and Earth-fixed reference frame, which violates the equivalence of all inertial frames [25]. Moreover, a virtual experiment was carried out by analyzing the raw GPS data using the parameter $\delta c/c$, where c is the round-trip speed of light, and δc is the deviation from c of the observed velocity of a light signal traveling one way along a particular spatial direction with the measuring clocks synchronized using slow clock transport [26]. In special relativity, $\delta c/c = 0$, but the experimental results show that the value $\delta c/c$ is a function of the direction of the velocity of clocks and the extremum value of $\delta c/c$ is 4.9×10^{-9} at a colatitude of 2.9 rad and a longitude of 0.5 rad (the raw data were based on six days: September 18–23, 1994). Further analysis in this study revealed that the value of $\delta c/c$ is related to Earth's velocity in relation to the "universal frame" through the equation $\delta c/c = (1 + 2\alpha)v/c$. Although the authors of this paper concluded that this experiment cannot suggest a violation of special relativity because the order of magnitude of the systematic effects is larger than the maximum observed $\delta c/c$ value, this experimental result is consistent with the concept of absolute space. It is expected that data covering more than one year can provide better results. In conclusion, the motional frequency shift in the clock in the GPS supports the existence of special relativity effects owing to the intrinsic properties of moving matter.

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